Hadamard well-posedness of multiobjective optimization problems

Rubén López*
Departamento de Matemática
Universidad de Tarapacá
Arica, Chile

Resumen

When studying the correctness of formulations of problems of mathematical physics, Hadamard has introduced in [3] a notion of well-posedness. He considered that as problems of mathematical physics describe real physical processes, their mathematical formulations must satisfy the following natural requirements:

(a) the solution must exist within a class of functions C_1;

(b) the solution must be unique within a class of functions C_2;

(c) the solution must depend continuously on the data of the problem.

We say that a problem is *Hadamard well-posed* if satisfies the requirements (a)-(c). The set of functions $C_1 \cap C_2$ is called the Hadamard well-posedness class. By dropping the uniqueness condition we obtain the notion of generalized Hadamard well-posed. The existence of solutions postulated in (a) indicates that the model is coherent and the uniqueness and stability postulated in (b)-(c) facilitate the development of accurate numerical approximations.

In this work, we study the well-posedness of families of finite dimensional multiobjective optimization problems. For these problems, two Hadamard well-posedness concepts are introduced. These concepts involve the existence and uniqueness of efficient/weak efficient solutions, and also the continuous behavior of these solutions with respect to perturbations of the data. The perturbations in the last property are formulated through a variational convergence notion of the objective functions (see [4, 5]) and by considering approximate solutions of the perturbed problems (see [1]). Necessary and sufficient conditions for the well-posedness of Pareto optimization problems are obtained in general, and also under convexity and quasiconvexity assumptions. Finally, it is proved that the convex multiobjective optimization problems are "essentially" well-posed in the sense of category theory.

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Referencias

[1] C. Gutiérrez, R. López, V. Novo, *Generalized ε-quasi solutions in multiobjective optimization problems: existence and optimality results*. Nonlinear Anal. TMA, Vol. 72, 2010, 4331–4346.

[2] C. Gutiérrez, R. López, V. Novo, *On Hadamard well-posedness of families of Pareto optimization problems*. J. Math. Anal. Appl., Vol. 444, 2016, 881–899.

[3] J. Hadamard, *Sur les problèmes aux dérivés partielles et leur signification physique*, Princeton University Bulletin, 13 (1902), 49–52.

[4] López R., Vera C., *On the set of weakly efficient minimizers for convex multiobjective programming*, Oper. Res. Lett., Vol. 36, 2008, 651–655.

[5] López R., *Variational convergence for vector valued-functions and its applications to convex multiobjective optimization*. Math. Meth. Oper. Res., Vol. 78, 2013, 1–34.

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